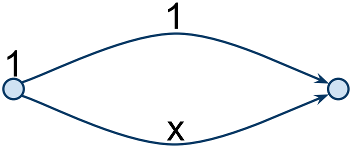
Emergency units and time dependent queueing games

@drvinceknight

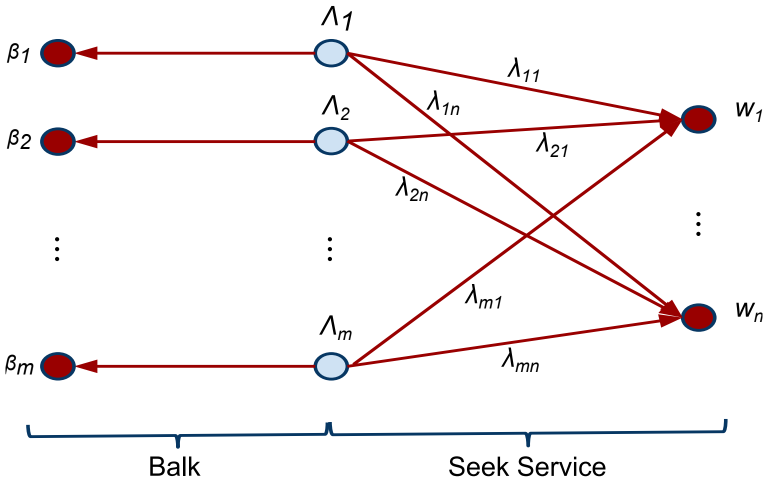
$$\begin{pmatrix} (2,2) & (5,0) \\ (0,5) & (4,4) \end{pmatrix}$$

- $k = 1$
- $\mathcal{P}_1 = \{1, 2\}$
- $c_1 = 1$ and $c_2 = x$
- $r = 1$



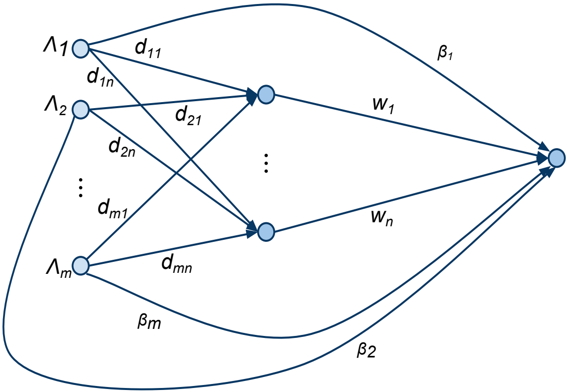
The Nash flow minimises:

$$\Phi(y, 1-y) = \sum_{e=1}^{2} \int_0^{f_e} c_e(x)dx = \int_0^{y} 1dx + \int_0^{1-y} xdx$$
$$= y + \frac{(1-y)^2}{2} = \frac{1}{2} + \frac{y^2}{2}$$
$$\Rightarrow \tilde{f} = (0, 1)$$



Λ_1
β_1
λ_{11}
λ_{1n}
Λ_2
w_1
β_2
λ_{21}
λ_{2n}
λ_{m1}
Λ_m
w_n
β_m
λ_{mn}
Balk
Seek Service

- $k = m$
- $|\mathcal{P}_i| = n + 1$
- $r_i = \Lambda_i$



- $k = m$
- $|\mathcal{P}_i| = n + 1$
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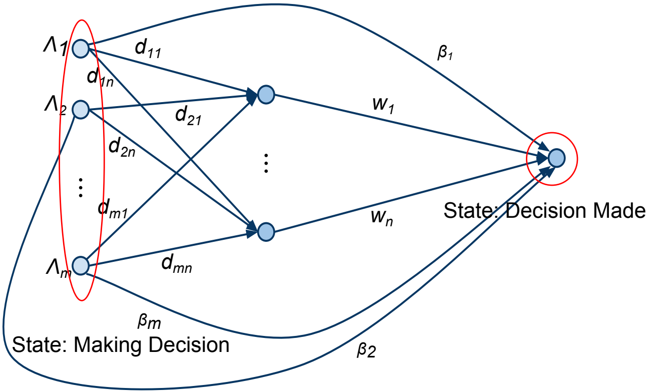


Theorem Assuming $\sum_{i=1}^{m} \Lambda_i < \sum_{j=1}^{n} c_j \mu_j$ we have:

$$\lim_{\beta_i \to \infty} PoA(\beta) < \infty \text{ for all } i \in [m]$$

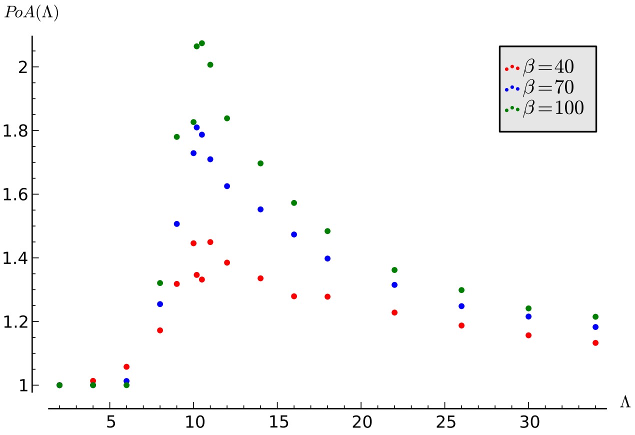
The price of anarchy increases with worth of service, up to a point.

Proof.

- $\lim_{\beta_i \to \infty} \lambda^* = k^*$ and $\lim_{\beta_i \to \infty} \tilde{\lambda} = \tilde{k}$
- As $\beta_i \to \infty$:

$$\sum_{i=1}^{m} \Lambda_i = \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_{ij}^* = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{\lambda}_{ij}$$

- $PoA(\beta) < \infty$



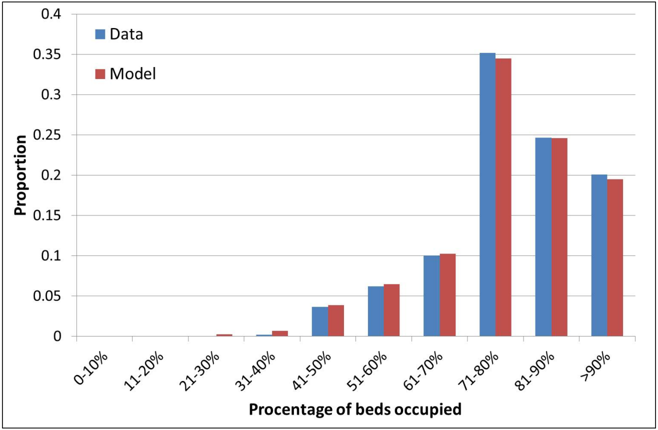
$PoA(\Lambda)$
2
1.8
1.6
1.4
1.2
1
5
10
15
20
25
30
Λ
$\beta = 40$
$\beta = 70$
$\beta = 100$

Price of Anarchy in Public Services *EJORS*, 2013.

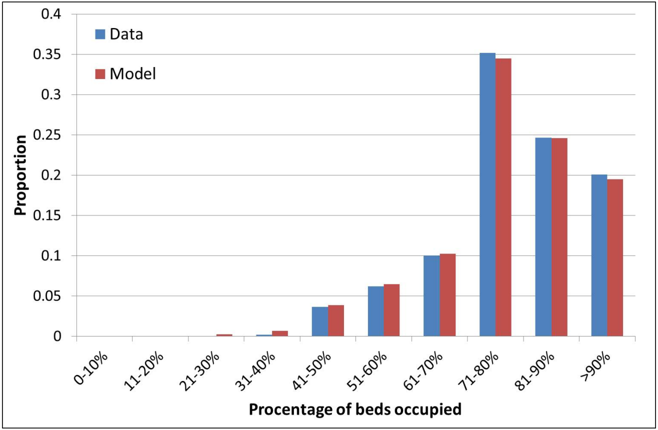
What about the controllers?

What about the controllers?

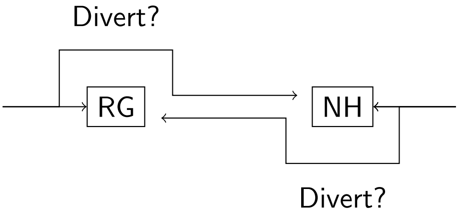
S. Deo and I. Gurvich. **Centralized vs. Decentralized Ambulance Diversion: A Network Perspective.** *Management Science*, May 2011.

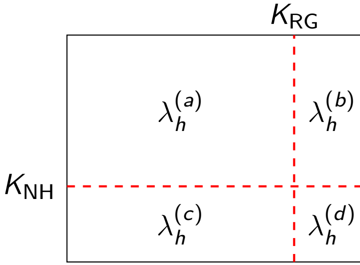


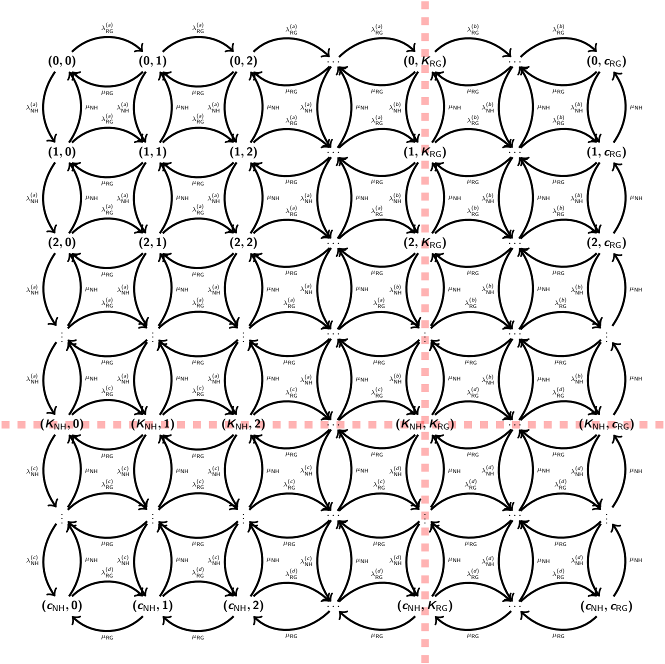
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Data
Model
Proportion
0-10%
11-20%
21-30%
31-40%
41-50%
51-60%
61-70%
71-80%
81-90%
>90%
Procentage of beds occupied

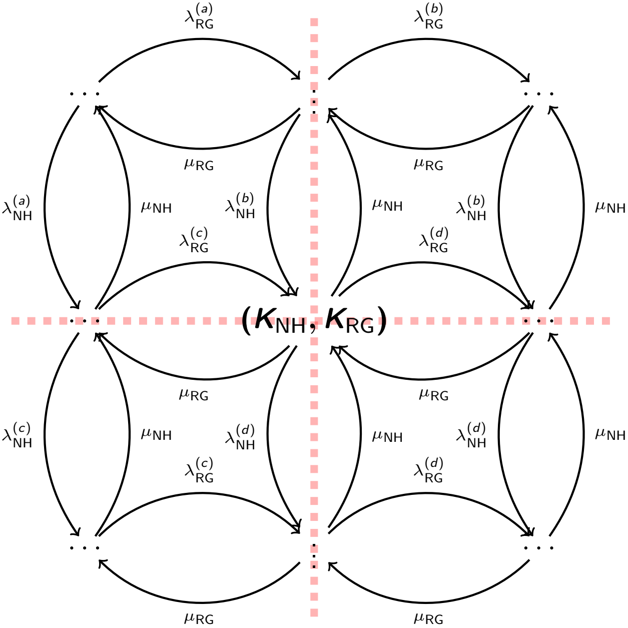


Mathematical modelling of patient flows to predict critical care capacity required following the merger of two District General Hospitals into one., *Anaesthesia*


Divert?
RG
NH
Divert?


K_{RG}
K_{NH}
$\lambda_h^{(a)}$
$\lambda_h^{(b)}$
$\lambda_h^{(c)}$
$\lambda_h^{(d)}$





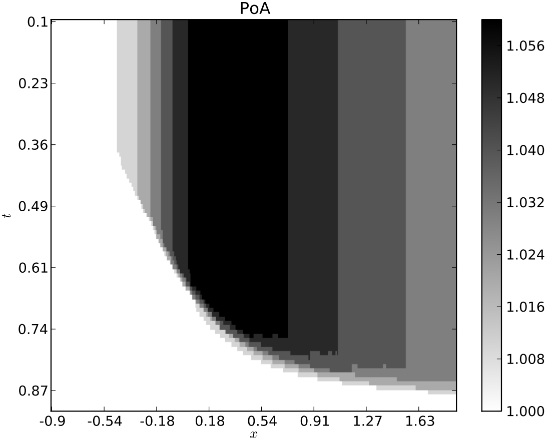
$\lambda_{RG}^{(a)}$
$\lambda_{RG}^{(b)}$
μ_{RG}
μ_{RG}
$\lambda_{NH}^{(a)}$
μ_{NH}
$\lambda_{NH}^{(b)}$
μ_{NH}
$\lambda_{NH}^{(b)}$
μ_{NH}
$\lambda_{RG}^{(c)}$
$\lambda_{RG}^{(d)}$
(K_{NH}, K_{RG})
μ_{RG}
μ_{RG}
$\lambda_{NH}^{(c)}$
μ_{NH}
$\lambda_{NH}^{(d)}$
μ_{NH}
$\lambda_{NH}^{(d)}$
μ_{NH}
$\lambda_{RG}^{(c)}$
$\lambda_{RG}^{(d)}$
μ_{RG}
μ_{RG}

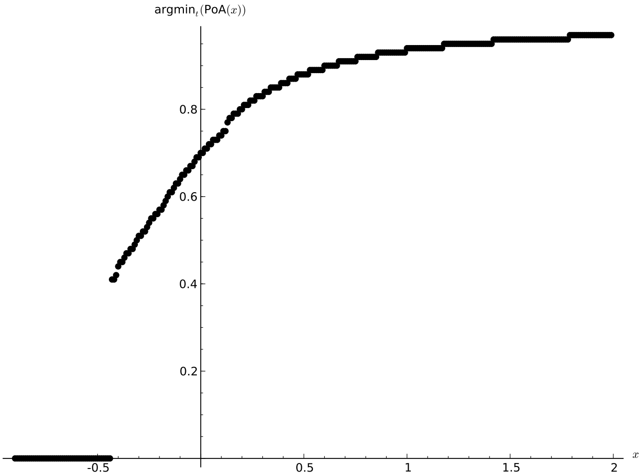
$$A = \begin{pmatrix} (U_{\mathsf{NH}}(1,1)-t)^2 & \dots & (U_{\mathsf{NH}}(1,c_{\mathsf{RG}})-t)^2 \\ (U_{\mathsf{NH}}(2,1)-t)^2 & \dots & (U_{\mathsf{NH}}(2,c_{\mathsf{RG}})-t)^2 \\ \vdots & \ddots & \vdots \\ (U_{\mathsf{NH}}(c_{\mathsf{NH}},1)-t)^2 & \dots & (U_{\mathsf{NH}}(c_{\mathsf{NH}},c_{\mathsf{RG}})-t)^2 \end{pmatrix}$$

$$B = \begin{pmatrix} (U_{\mathsf{RG}}(1,1)-t)^2 & \dots & (U_{\mathsf{RG}}(1,c_{\mathsf{RG}})-t)^2 \\ (U_{\mathsf{RG}}(2,1)-t)^2 & \dots & (U_{\mathsf{RG}}(2,c_{\mathsf{RG}})-t)^2 \\ \vdots & \ddots & \vdots \\ (U_{\mathsf{RG}}(c_{\mathsf{RG}},1)-t)^2 & \dots & (U_{\mathsf{RG}}(c_{\mathsf{RG}},c_{\mathsf{RG}})-t)^2 \end{pmatrix}$$

Theorem.

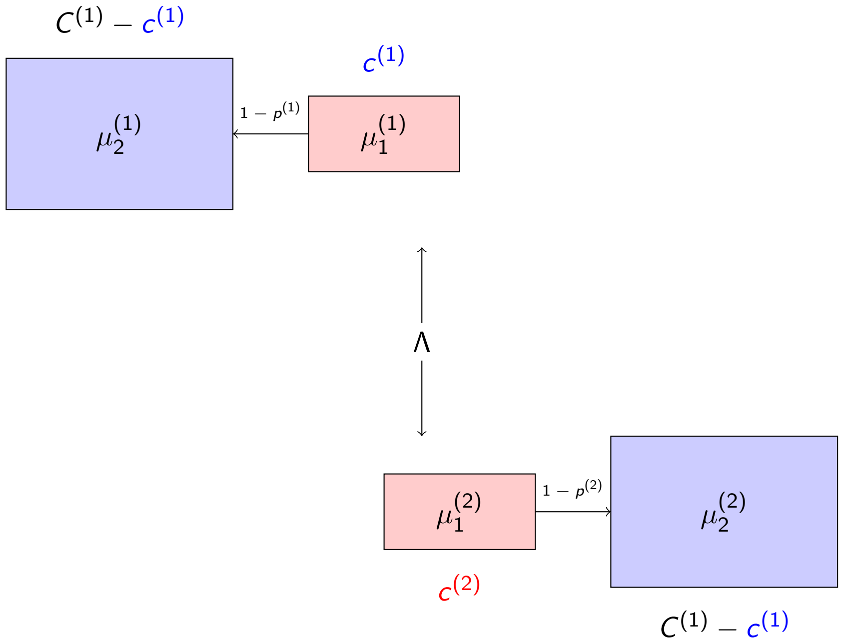
Let $f_h(k) : [1, c_{\bar{h}}] \to [1, c_h]$ be the best response of player $h \in \{\mathrm{NH}, \mathrm{RG}\}$ to the diversion threshold of $\bar{h} \neq h$ ($\bar{h} \in \{\mathrm{NH}, \mathrm{RG}\}$). If $f_h(k)$ is a non-decreasing function in k then the game has at least one Nash Equilibrium in Pure Strategies.


PoA
t
0.1
0.23
0.36
0.49
0.61
0.74
0.87
x
-0.9
-0.54
-0.18
0.18
0.54
0.91
1.27
1.63
1.056
1.048
1.040
1.032
1.024
1.016
1.008
1.000

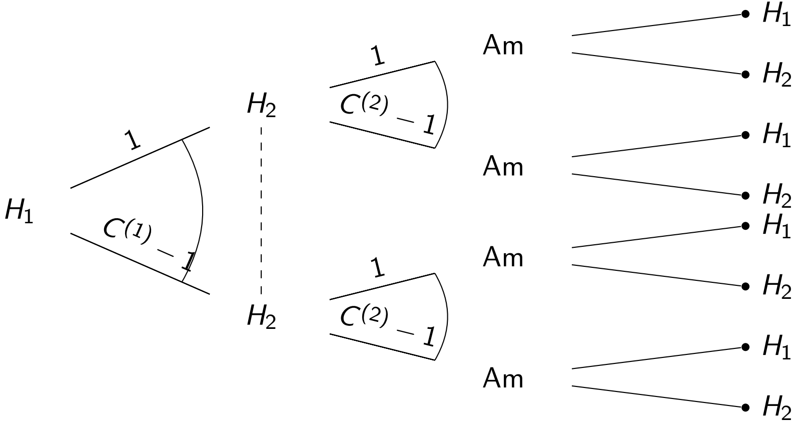

argmin$_t$(PoA(x))
0.8
0.6
0.4
0.2
-0.5
0.5
1
1.5
2
x

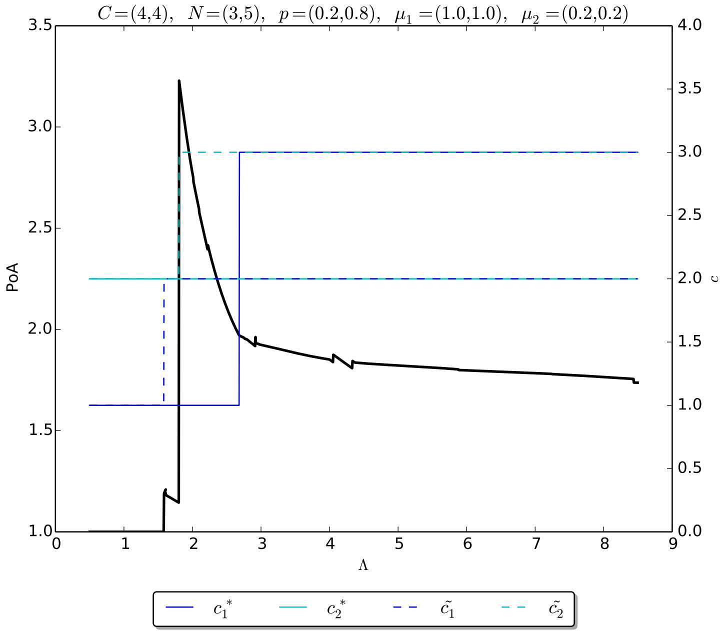
Measuring the Price of Anarchy in Critical Care Unit Interactions, *Journal of Operational Research*

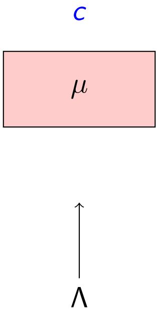

UNED ACHOSION BRYS
EMERGENCY UNIT
AMBULANCE
AMBULANCE

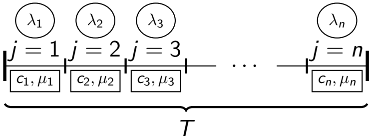


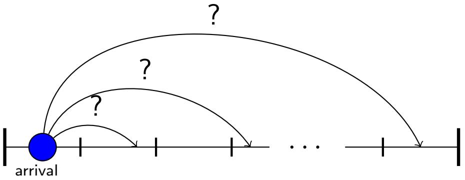
$C^{(1)} - c^{(1)}$
$c^{(1)}$
$\mu_2^{(1)}$
$1 - p^{(1)}$
$\mu_1^{(1)}$
Λ
$\mu_1^{(2)}$
$1 - p^{(2)}$
$\mu_2^{(2)}$
$c^{(2)}$
$C^{(1)} - c^{(1)}$

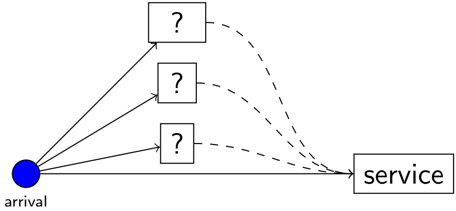

H_1
1
$C^{(1)} - 1$
H_2
H_2
1
$C^{(2)} - 1$
1
$C^{(2)} - 1$
Am
Am
Am
Am
H_1
H_2
H_1
H_2
H_1
H_2
H_1
H_2

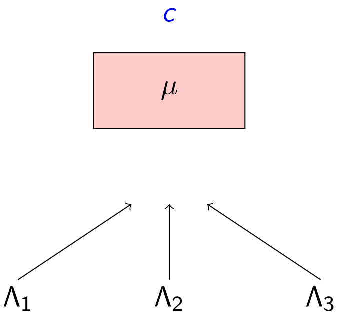

$C=(4,4), \quad N=(3,5), \quad p=(0.2,0.8), \quad \mu_1=(1.0,1.0), \quad \mu_2=(0.2,0.2)$
PoA
Λ
c
c_1^*
c_2^*
$\tilde{c}_1$
$\tilde{c}_2$


c
μ
Λ

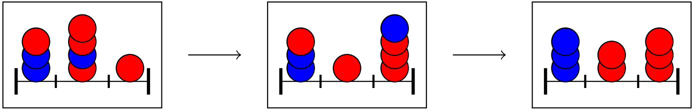

λ_1
λ_2
λ_3
λ_n
$j = 1$
$j = 2$
$j = 3$
$\cdots$
$j = n$
c_1, μ_1
c_2, μ_2
c_3, μ_3
c_n, μ_n
T


?
?
?
. . .
arrival

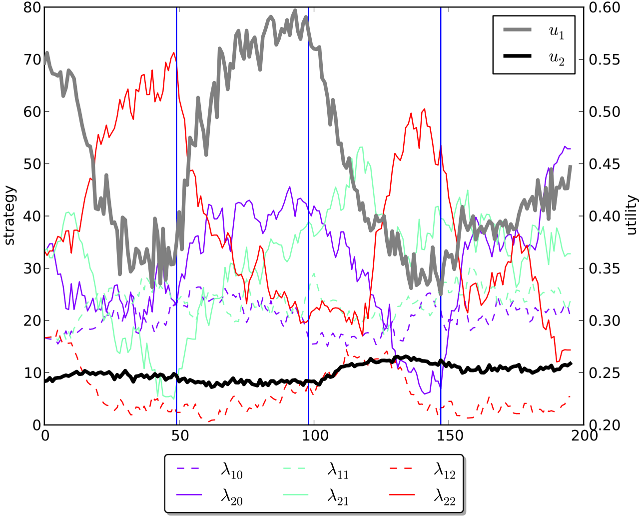

?
?
?
service
arrival



c
μ
Λ_1
Λ_2
Λ_3



Generate population for all sources
while *clock* $<$ *simulation time* **do**
    Randomly group agents from each source
    Run a simulation of process
    Identify worst performing agents
    Reproduce agents with mutation
end


strategy
utility
0
10
20
30
40
50
60
70
80
0
50
100
150
200
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
u_1
u_2
λ_{10}
λ_{11}
λ_{12}
λ_{20}
λ_{21}
λ_{22}

+Vincent.Knight

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