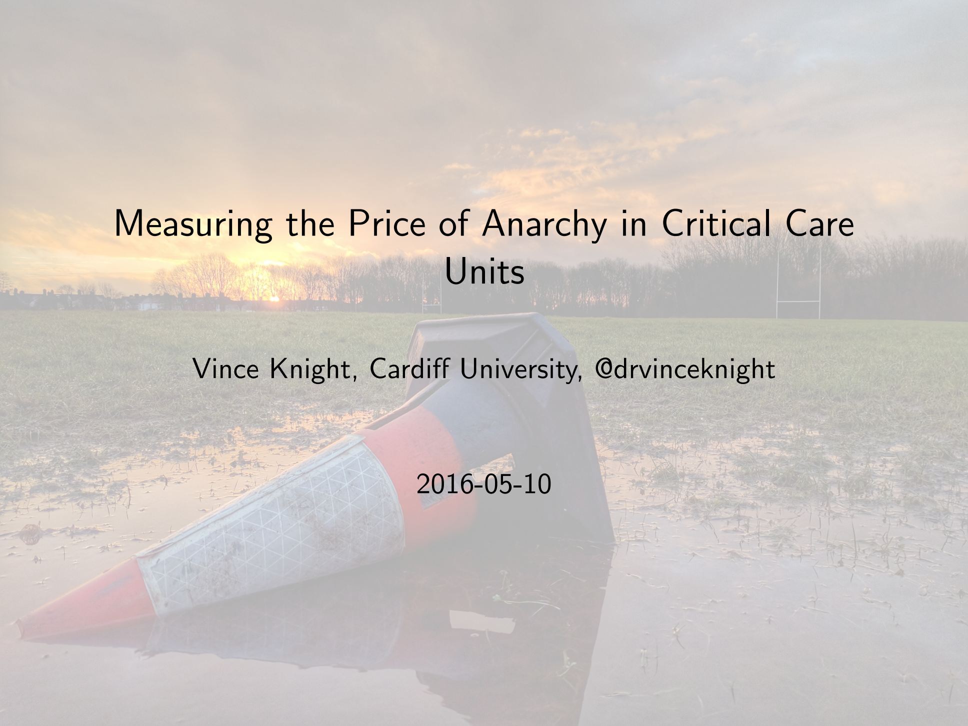
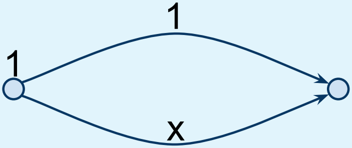


Measuring the Price of Anarchy in Critical Care Units

Vince Knight, Cardiff University, @drvinceknight

2016-05-10

- $k = 1$
- $\mathcal{P}_1 = \{1, 2\}$
- $c_1 = 1$ and $c_2 = x$
- $r = 1$



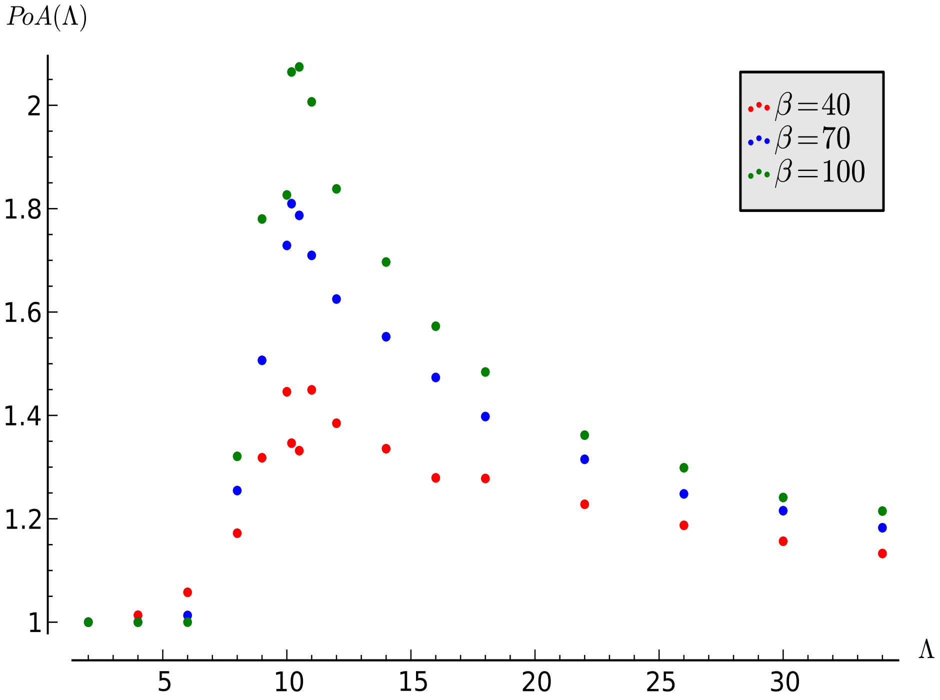
The Nash flow minimises:

$$\Phi(y, 1-y) = \sum_{e=1}^{2} \int_0^{f_e} c_e(x)dx = \int_0^{y} 1dx + \int_0^{1-y} xdx$$

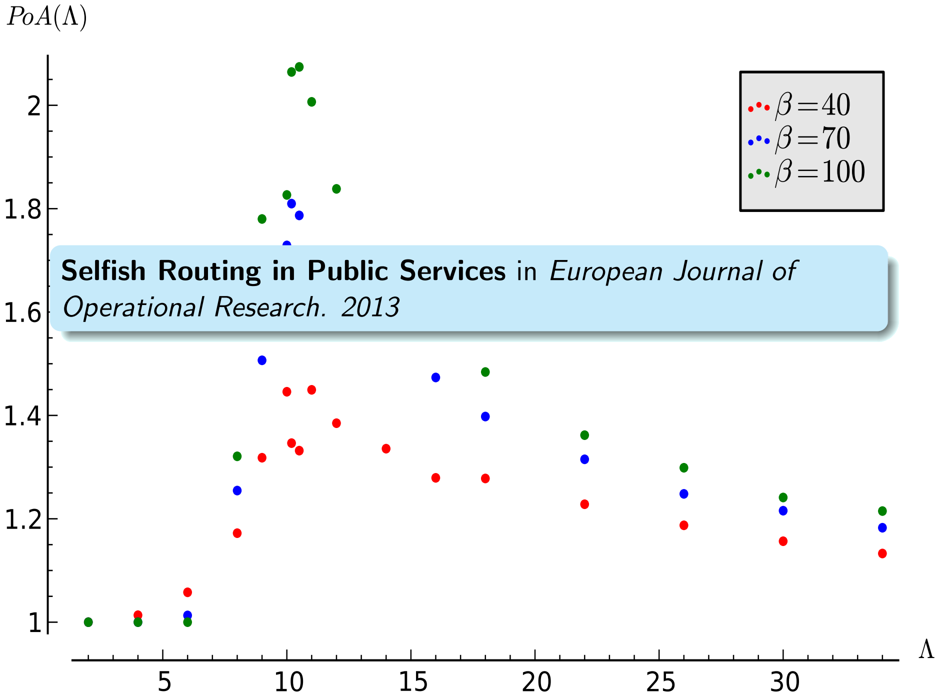
$$= y + \frac{(1-y)^2}{2} = \frac{1}{2} + \frac{y^2}{2}$$

$$\Rightarrow \tilde{f} = (0, 1)$$

Game Theory and Healthcare



$PoA(\Lambda)$
2
1.8
1.6
1.4
1.2
1
5
10
15
20
25
30
Λ
$\beta = 40$
$\beta = 70$
$\beta = 100$

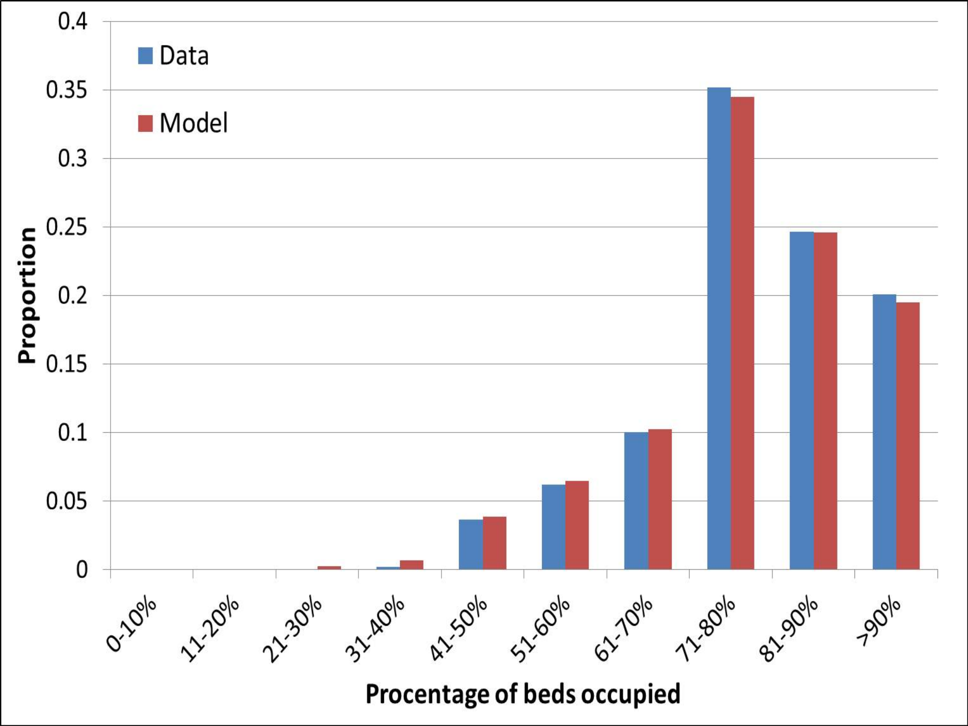


Selfish Routing in Public Services in *European Journal of Operational Research. 2013*

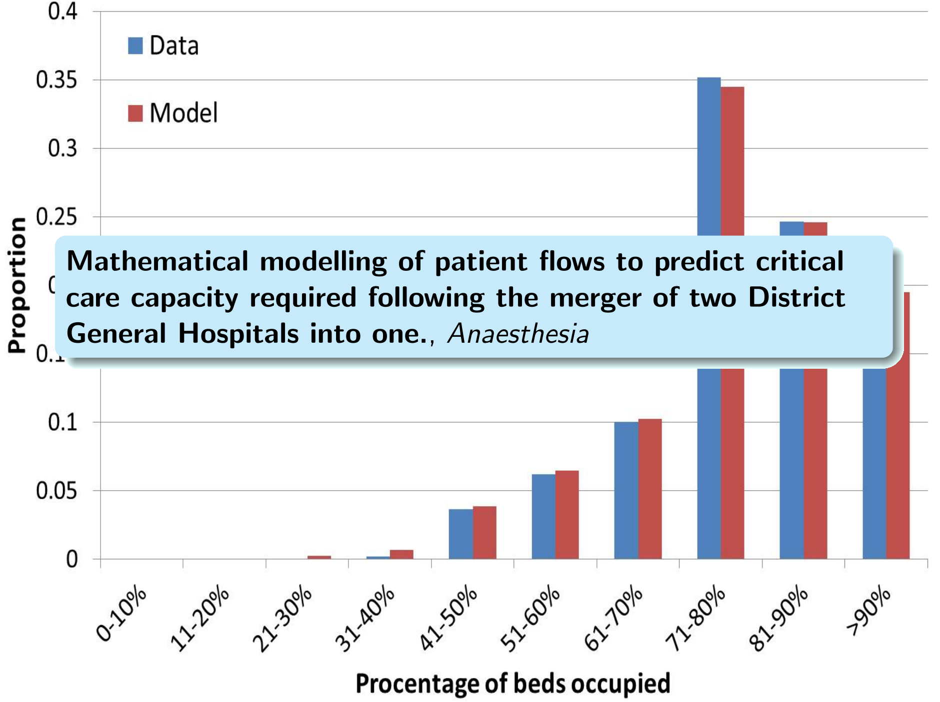
What about the controllers?

What about the controllers?

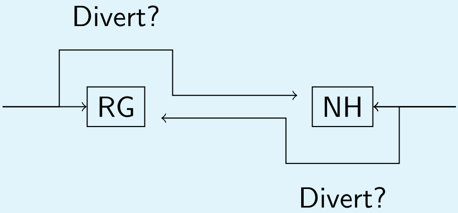
S. Deo and I. Gurvich. **Centralized vs. Decentralized Ambulance Diversion: A Network Perspective.** *Management Science*, 57(7):13001319, May 2011.

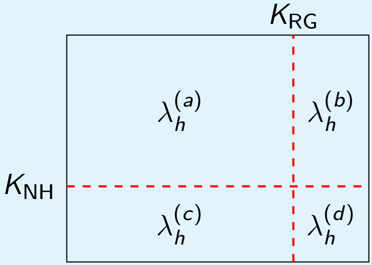


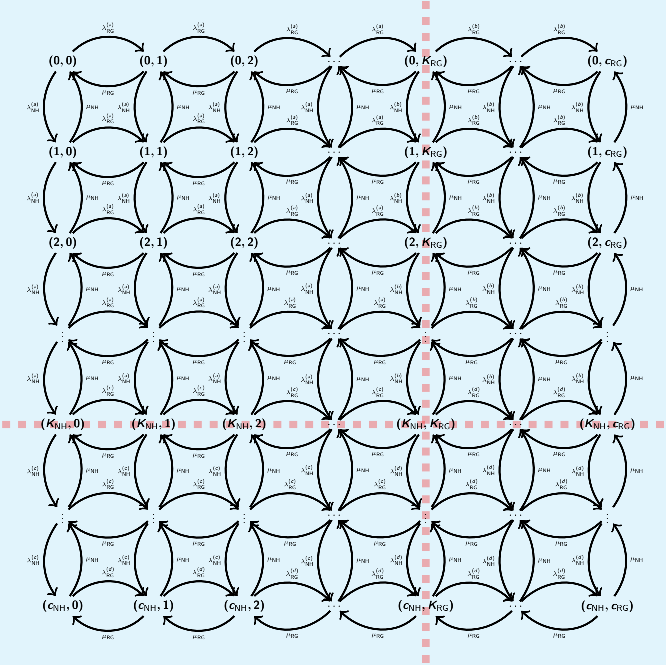
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Data
Model
Proportion
0-10%
11-20%
21-30%
31-40%
41-50%
51-60%
61-70%
71-80%
81-90%
>90%
Procentage of beds occupied

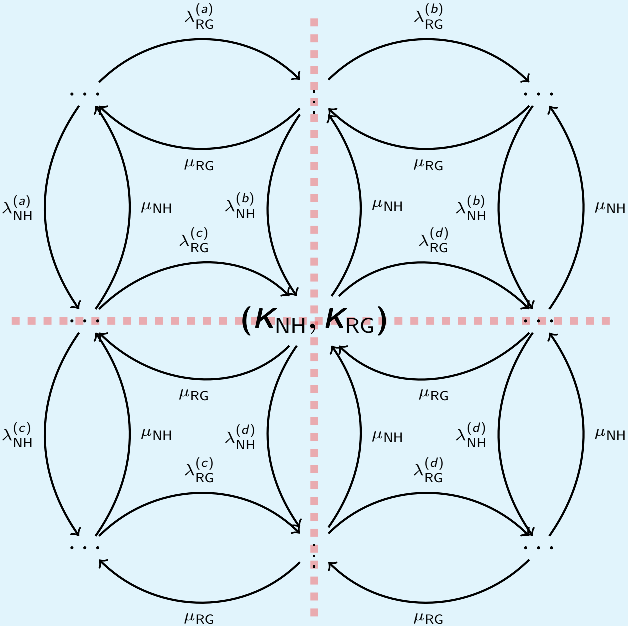


Mathematical modelling of patient flows to predict critical care capacity required following the merger of two District General Hospitals into one., *Anaesthesia*

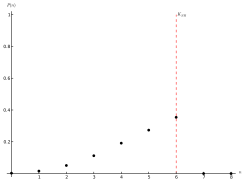

Divert?
RG
NH
Divert?


K_{RG}
$\lambda_h^{(a)}$
$\lambda_h^{(b)}$
K_{NH}
$\lambda_h^{(c)}$
$\lambda_h^{(d)}$

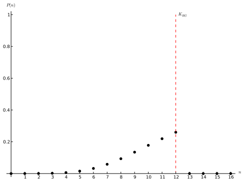

(0, 0)
(0, 1)
(0, 2)
(0, K_RG)
(0, c_RG)
(1, 0)
(1, 1)
(1, 2)
(1, K_RG)
(1, c_RG)
(2, 0)
(2, 1)
(2, 2)
(2, K_RG)
(2, c_RG)
(K_NH, 0)
(K_NH, 1)
(K_NH, 2)
(K_NH, K_RG)
(K_NH, c_RG)
(c_NH, 0)
(c_NH, 1)
(c_NH, 2)
(c_NH, K_RG)
(c_NH, c_RG)


$\lambda_{RG}^{(a)}$
$\lambda_{RG}^{(b)}$
μ_{RG}
μ_{RG}
$\lambda_{NH}^{(a)}$
μ_{NH}
$\lambda_{NH}^{(b)}$
μ_{NH}
$\lambda_{NH}^{(b)}$
μ_{NH}
$\lambda_{RG}^{(c)}$
$\lambda_{RG}^{(d)}$
(K_{NH}, K_{RG})
μ_{RG}
μ_{RG}
$\lambda_{NH}^{(c)}$
μ_{NH}
$\lambda_{NH}^{(d)}$
μ_{NH}
$\lambda_{NH}^{(d)}$
μ_{NH}
$\lambda_{RG}^{(c)}$
$\lambda_{RG}^{(d)}$
μ_{RG}
μ_{RG}

$(K_{\mathsf{NH}}, K_{\mathsf{RG}}) = (6, 12)$:

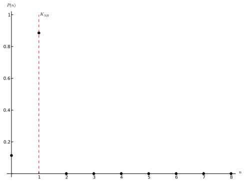


$h = NH$

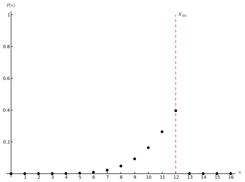


$h = RG$

$(K_{\mathsf{NH}}, K_{\mathsf{RG}}) = (1, 12)$:



$h = \mathrm{NH}$



$h = \mathrm{RG}$

For all $h \in \{\mathrm{NH}, \mathrm{RG}\}$ minimise:

$$(U_h - t)^2$$

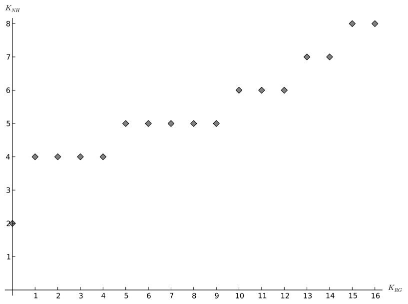
Subject to:

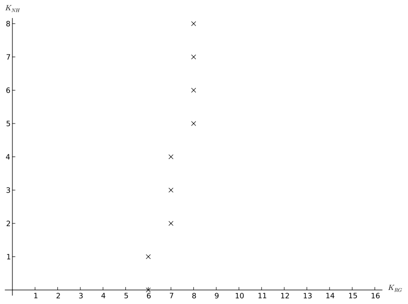
$$0 \leq K_h \leq c_h$$
$$K_h \in \mathbb{Z}$$

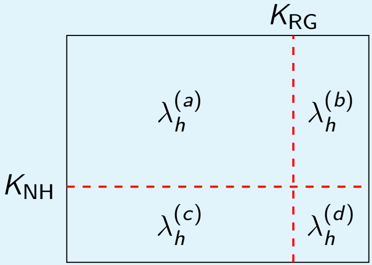
$$A = \begin{pmatrix} (U_{\mathsf{NH}}(1,1)-t)^2 & \dots & (U_{\mathsf{NH}}(1,c_{\mathsf{RG}})-t)^2 \\ (U_{\mathsf{NH}}(2,1)-t)^2 & \dots & (U_{\mathsf{NH}}(2,c_{\mathsf{RG}})-t)^2 \\ \vdots & \ddots & \vdots \\ (U_{\mathsf{NH}}(c_{\mathsf{NH}},1)-t)^2 & \dots & (U_{\mathsf{NH}}(c_{\mathsf{NH}},c_{\mathsf{RG}})-t)^2 \end{pmatrix}$$

$$B = \begin{pmatrix} (U_{\mathsf{RG}}(1,1)-t)^2 & \dots & (U_{\mathsf{RG}}(1,c_{\mathsf{RG}})-t)^2 \\ (U_{\mathsf{RG}}(2,1)-t)^2 & \dots & (U_{\mathsf{RG}}(2,c_{\mathsf{RG}})-t)^2 \\ \vdots & \ddots & \vdots \\ (U_{\mathsf{RG}}(c_{\mathsf{RG}},1)-t)^2 & \dots & (U_{\mathsf{RG}}(c_{\mathsf{RG}},c_{\mathsf{RG}})-t)^2 \end{pmatrix}$$

Theorem.
Let $f_h(k) : [1, c_{\bar{h}}] \to [1, c_h]$ be the best response of player $h \in \{\mathrm{NH}, \mathrm{RG}\}$ to the diversion threshold of $\bar{h} \neq h$ ($\bar{h} \in \{\mathrm{NH}, \mathrm{RG}\}$). If $f_h(k)$ is a non-decreasing function in k then the game has at least one Nash Equilibrium in Pure Strategies.

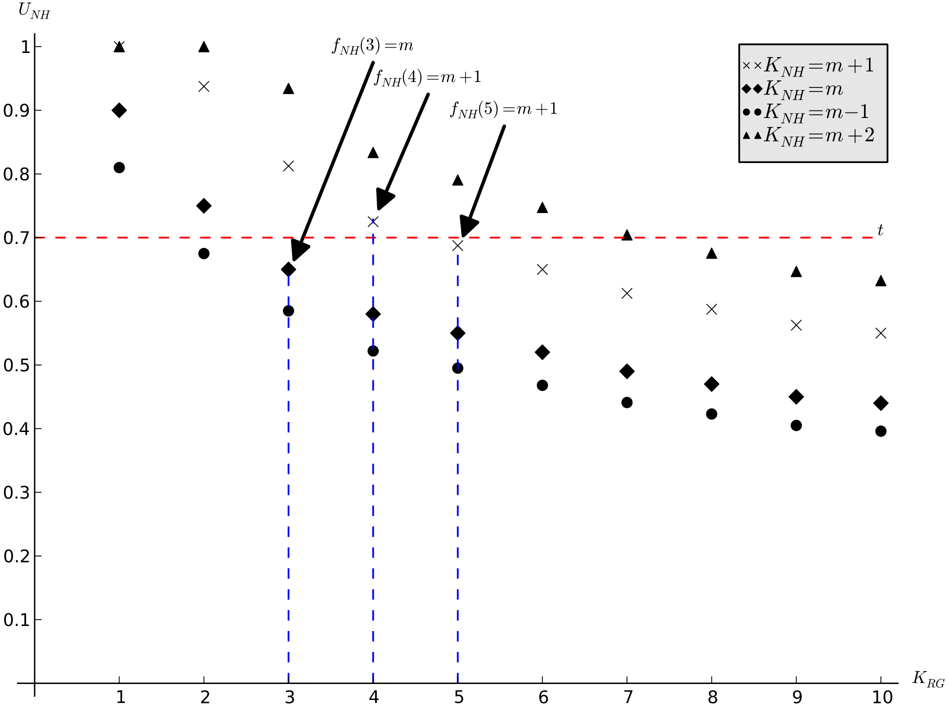

K_{NH}
K_{RG}
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16


K_{NH}
K_{RG}
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

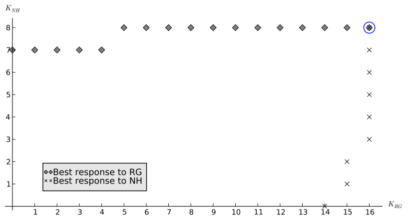


Lemma.

- If $\lambda_{\text{NH}}^{(a)} \leq \lambda_{\text{NH}}^{(b)}$ and $\lambda_{\text{NH}}^{(c)} \leq \lambda_{\text{NH}}^{(d)}$ then $f_{\text{NH}}(k)$ is a non-decreasing function in k.
- If $\lambda_{\text{RG}}^{(a)} \leq \lambda_{\text{RG}}^{(c)}$ and $\lambda_{\text{RG}}^{(b)} \leq \lambda_{\text{RG}}^{(d)}$ then $f_{\text{RG}}(k)$ is a non-decreasing function in k.

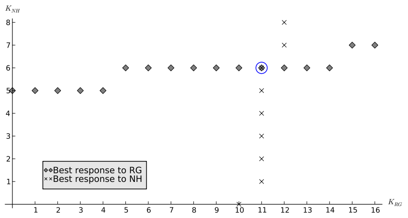


U_{NH}
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$f_{NH}(3)=m$
$f_{NH}(4)=m+1$
$f_{NH}(5)=m+1$
$K_{NH}=m+1$
$K_{NH}=m$
$K_{NH}=m-1$
$K_{NH}=m+2$
t
1 2 3 4 5 6 7 8 9 10
K_{RG}



K_{NH}
K_{RG}
Best response to RG
Best response to NH

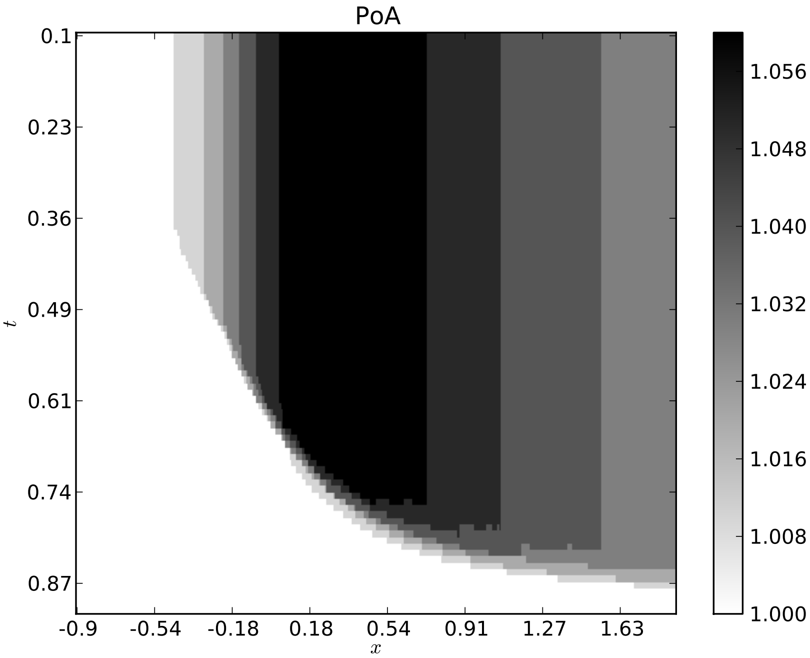
$(t = 0.8)$

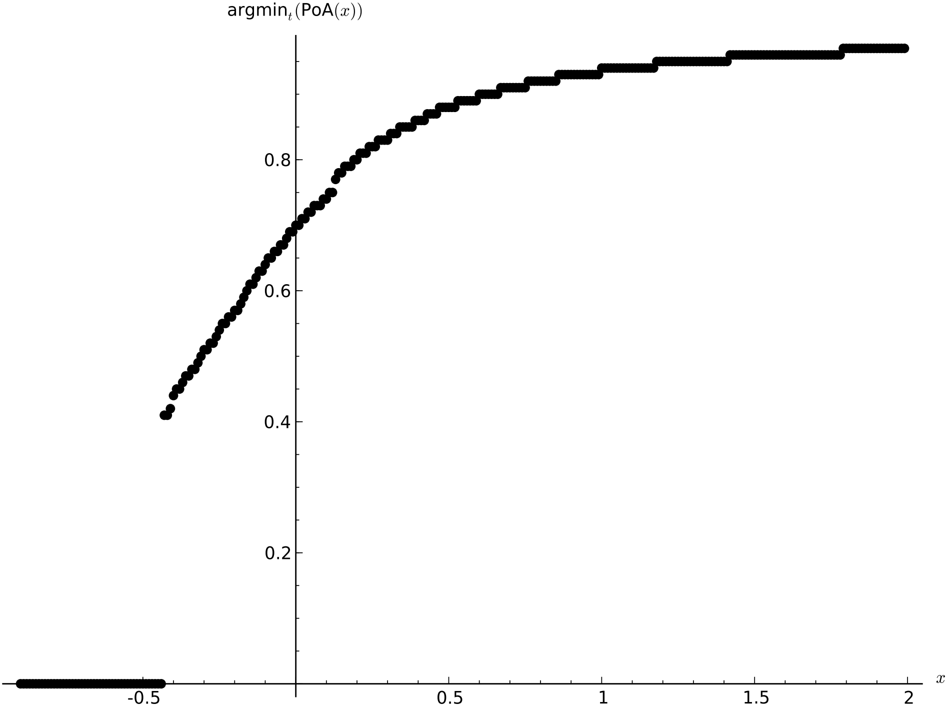


K_{NH}
K_{RG}
Best response to RG
Best response to NH

$(t = 0.6)$

$$\mathrm{PoA} = \frac{T^*}{\widetilde{T}}$$


PoA
0.1
0.23
0.36
0.49
0.61
0.74
0.87
t
-0.9
-0.54
-0.18
0.18
0.54
0.91
1.27
1.63
x
1.056
1.048
1.040
1.032
1.024
1.016
1.008
1.000



argmin$_t$(PoA(x))
0.8
0.6
0.4
0.2
-0.5
0.5
1
1.5
2
x

Conclusions

- Developed a strategic form game representation of CCU interaction;
- Proved structural properties of equilibrium behaviour;
- Identified a potential justified approach to obtaining policies.

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Measuring the Price of Anarchy in Critical Care Unit Interactions, *Minor revisions to Journal of Operational Research*

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github.com/Axelrod-Python/Axelrod