Computing for Mathematics: Handout 9

This handout contains a summary of the topics covered and an activity to carry out prior or during your lab session.

At the end of the handout is a specific coursework like exercise.

For further practice you can do the exercises available at the differential equations chapter of Python for Mathematics.

1 Summary

The purpose of this handout is to cover differential equations which corresponds to the differential equations chapter of Python for Mathematics.

The topics covered are:

- Creating a symbolic function
- Writing a differential equation
- Solving a differential equation

2 Activity

We will be tackling the problem from the tutorial of the differential equations chapter of Python for Mathematics.

A container has volume V of liquid which is poured in at a rate proportional to e^{-t} (where t is some measurement of time). Initially the container is empty and after $t = 3$ time units the rate at which the liquid is poured is 15.

1. Show that $V(t) = \frac{-15e^3}{1-e^3}(1 - e^{-t})$ 2. Obtain the limit $\lim_{t\to\infty} V(t)$

There are instructions for how to do all of this is in the differential equations chapter of Python for Mathematics.

1. Create the symbolic variables `t` and `k` as well as the symbolic function `V`.
2. Create the variable `differential_equation` which has value the differential equation $\frac{d}{dt}V(t) = ke^{-t}$
3. Use the `sympy.dsolve` tool to obtain the general solution to this differential equation.
4. Use the initial conditions given (that $V(0) = 0$) to obtain the particular solution to this differential equation.
5. Use the fact that $V(3) = 15$ to obtain a particular value for k.
6. Obtain the required limit.

3 Coursework like exercise

1. Create a variable `differential_equation` that has value a the differential equation: $\frac{dy}{dx} = \cos y$.
2. Create a variable `general_solution` that has value the general solution (as an equation) for the differential equation.
3. Create a variable `particular_solution` that has value the particular solution (as an equation) for the differential equation for the condition that $y(\pi) = 5$.

4 Summary examples

Create a symbolic function g:

```
import sympy as sym
g = sym.Function(g)
```

Create the differential equation $\frac{dy}{dx} = x$:

```
import sympy as sym
y = sym.Function(g)
x = sym.Function(x)

lhs = sym.diff(y(x), x)
differential_equation = sym.Eq(lhs, x)
```

Solve the differential equation $\frac{dy}{dx} = x^2$ given the condition $y(1) = 0$

```
import sympy as sym
y = sym.Function(g)
x = sym.Function(x)

lhs = sym.diff(y(x), x)
rhs = x ** 2
differential_equation = sym.Eq(lhs, rhs)

condition = {y(1): 0}
solution = sym.dsolve(
    differential_equation,
    y(x),
    ics=condition,
)
```