Computing for Mathematics: Handout 6

This handout contains a summary of the topics covered and an activity to carry out prior or during your lab session.

At the end of the handout is a specific coursework like exercise.

For further practice you can do the exercises available at the matrices chapter of Python for Mathematics.

1 Summary

The purpose of this handout is to cover matrices which corresponds to the probability chapter of Python for Mathematics.

The topics covered are:

- Creating matrices.
- Manipulating matrices.
- Solving a system of linear equations using matrices.

2 Activity

We will be tackling the problem from the tutorial of the matrices chapter of Python for Mathematics.

The matrix A is given by $A = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 2 \end{pmatrix}$.

1. Find the determinant of A 2. Hence find the values of a for which A is singular. 3. For the following values of a, when possible obtain A^{-1} and confirm the result by computing AA^{-1}: 1. $a = 0$; 2. $a = 1$; 3. $a = 2$; 4. $a = 3$.

There are instructions for how to do all of this is in the probability chapter of Python for Mathematics.

1. Create a variable `A` which has value the matrix A.
2. Create a variable `determinant` which has value the determinant of A.
3. Find the values of a for which the determinant of A is 0. This corresponds to the values for which A is singular.
4. Substitute the given values of a in to `A` and compute the inverse. Multiply the inverse by A to obtain the identity matrix $\begin{pmatrix} 1 & 0 & \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ which confirms the result.

3 Coursework like exercise

The matrix A is given by $A = \begin{pmatrix} a & 3 & 1 \\ a & 2a & 3 \\ -3 & 2 & 2a \end{pmatrix}$.

1. Create a variable `determinant` which has value the determinant of A
2. Create a variable `singular_values_of_a` which has value the set of values of a for which A is singular.
3. Output a list which contains the singular values of a which are pure real numbers.

4 Summary examples

Create the matrix $B = \begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix}$.

```
import sympy as sym
B = sym.Matrix(((3, 5), (1, -2)))
```

Obtain the determinant of $B = \begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix}$.

```
import sympy as sym
B = sym.Matrix(((3, 5), (1, -2)))
B.det()
```

Obtain the inverse of $B = \begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix}$

```
import sympy as sym
B = sym.Matrix(((3, 5), (1, -2)))
B.inv()
```

Calculate $\begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix} \left(\begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix} + 6 \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} \right)$

```
import sympy as sym
B = sym.Matrix(((3, 5), (1, -2)))
C = sym.Matrix(((3, 1), (4, 1)))
D = sym.Matrix(((2, 3), (1, 1)))
B @ (C + 6 * D)
```

Solve the linear system:

$$
\begin{aligned}
x + 2y &= 3 \\
3x + y &= 4
\end{aligned}
$$

```
import sympy as sym
M = sym.Matrix(((1, 2), (3, 1)))
b = sym.Matrix(((3,), (4,)))
M.inv() @ b
```