Decision Theory Exercise Sheet Solutions

Last updated: October 6, 2012.

1. The optimal decisions are:
   - MaxMax: Vaccination
   - MaxMin: Cure
   - MinMax Regret: Vaccination and/or Cure (further analysis needed)
   - Max Likelihood: Vaccination
   - Max Expected Value: Vaccination

	Cure	No Cure	Max	Min	Max Regret	ML	Max. Exp.
Symptomatic relief	-10	30	30	-10	35	30	24
Vaccination	5	60	60	5	20	60	51.75
Cure	25	40	40	25	20	40	37.75

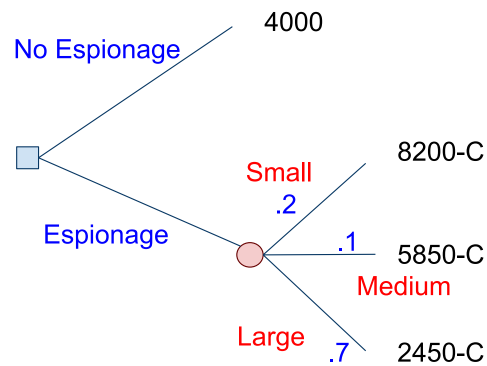
2. We have $P(S) = P(M) = P(L) = \frac{1}{3}$.
   - MaxMax: Large
   - MaxMin: Small
   - MinMax Regret: Medium and/or Large (further analysis needed)
   - Max Likelihood: Large
   - Max Expected Value: Medium

	Small	Medium	Large	Max	Min	Max Regret	ML	Max. Exp
Small	4000	3000	2000	4000	2000	5000	4000	3000
Medium	5000	6000	1000	6000	1000	4000	6000	4000
Large	9000	2000	0	9000	0	4000	9000	3667

3. If PS, PM and PL denotes the event that espionage predicts small, medium and large. Then, from the question we have $P(PS) = .2$, $P(PM) = .1$ and $P(PL) = .7$. Also:

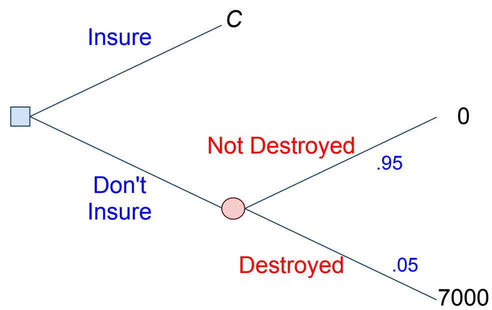
$$\begin{array}{ccc} P(S|PS) = .9 & P(S|PM) = .025 & P(S|PL) = .15 \\ P(M|PS) = .05 & P(M|PM) = .95 & P(M|PL) = .15 \\ P(L|PS) = .05 & P(L|PM) = .025 & P(L|PL) = .7 \end{array}$$

A reduced version of the tree is shown:



Thus the expected return for "Espionage" is $3940 - C$ and so "Espionage" is never the correct decision (unless C is negative).

4. A reduced version of the tree is shown:



It is worth (financially!) getting insurance if $\sqrt{C+1000} \leq .95\sqrt{1000} + .05\sqrt{8000}$ which reduces to $C \leq 191.201$.

5. A risk averse strategy implies that the required decision is to "not flip". Thus we need $4000^{\frac{1}{n}} \geq \frac{1}{2}10000^{\frac{1}{n}}$. Solving this inequality gives:

$$\frac{1}{n}\ln 4000 \geq \frac{1}{n}\ln 10000 - \ln 2$$
$$\frac{1}{n}\ln \frac{5}{2} \leq \ln 2$$
$$n \geq \frac{\ln \frac{5}{2}}{\ln 2} \approx 1.32$$

However since $n \in \mathbb{Z}$, this gives $n \geq 2$. From the above equation we see that the important factor is the ratio of the two payoffs. Importantly this mean that **millions** or **thousands** will give the same result.

6. Let:

   - D denote the event "purchase from dealer".
   - P denote the event "purchase privately".
   - AA denote the event "get the AA to check".
   - $NoAA$ denote the event "not using the AA".
   - F denote the event "the second hand car is faulty".
   - NoF denote the event "the second hand car is not faulty".
   - $AAfF$ denote the event "the AA finds a fault".
   - $AAdfF$ denote the event "the AA does not find a fault".

   From the question we have:

$$P(F) = P(NoF) = .5$$
$$P(AAfF \mid F) = .8$$
$$P(AAfF \mid NoF) = 0$$

thus:

$$
\begin{aligned}
P(AAfF) &= P(AAfF \mid F)P(F) + P(AAfF \mid NoF)P(NoF) = .8 \times .5 + 0 = \tfrac{2}{5} \\
P(AAdfF) &= 1 - P(AAfF) = \tfrac{3}{5} \\
P(F \mid AAfF) &= \tfrac{P(F)P(AAfF \mid F)}{P(AAfF)} = 1 \\
P(noF \mid AAfF) &= 0 \\
P(F \mid AAdfF) &= \tfrac{P(F)P(AAdfF \mid F)}{P(AAdfF)} = \tfrac{1}{6} \\
P(noF \mid AAdfF) &= \tfrac{5}{6}
\end{aligned}
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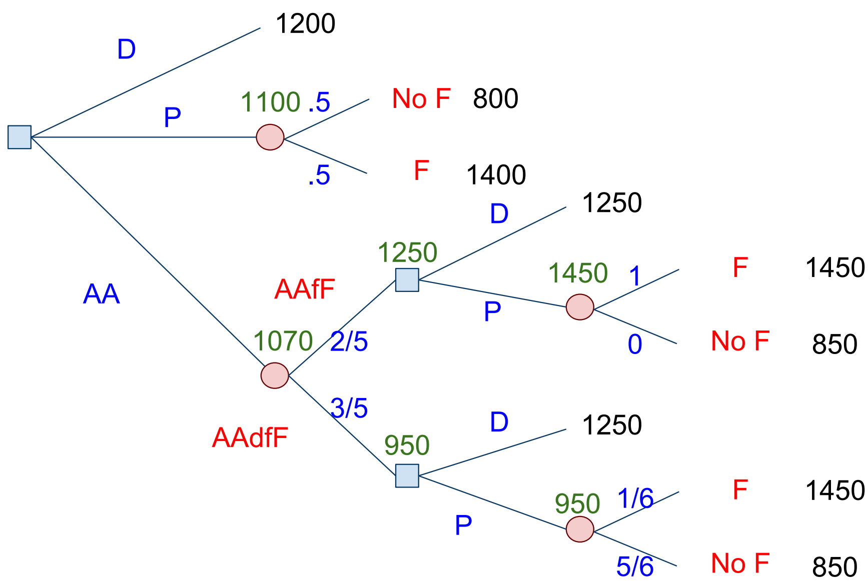

thus the best financial option is to seek the AAs advice.