

Decision Theory Exercise Sheet

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1. A large pharmaceutical company must decide on 3 possible alternatives for investment:

- Invest in the research of a drug that suppresses the symptoms of laughteritis (a disease that cause uncontrollable laughter).
- Invest in the research of a vaccination that prevent catching laughteritis.
- Invest in the more costly alternative of finding a drug that cures laughteritis.

There is a 15% chance that a rival company is able to find a drug that cures laughteritis. The returns on investment are given by the following table:

	Rival finds cure	Rival doesn't find cure
Invest in symptomatic relief	-10	30
Invest in vaccination	5	60
Invest in cure	25	40

Use the following 5 approaches to inform the pharmaceutical company's decision:

- MaxMax
- MaxMin
- MinMax Regret
- Maximum likelihood
- Maximum expected

2. Assume that 2 pizza companies: *Drafts* and *Pizza shack* are 2 competing firms. They are both about to embark on a *small*, *medium* or *large* advertising campaign. Pizza shack believes that it is equally likely that Drafts will undertake a *small*, *medium* or *large* advertising campaign. Given the actions chosen by each restaurant, Pizza shack's profits are given by the following table:

	D. chooses small	D. chooses medium	D. chooses large
P.s. chooses small	4000	3000	2000
P.s. chooses medium	5000	6000	1000
P.s. chooses large	9000	2000	0

Use the following five approaches to inform Pizza shack's decision:

- MaxMax
- MaxMin
- MinMax Regret
- Maximum likelihood
- Maximum expected

3. Using the previous example. Assume Pizza shack has the option of paying a fixed cost of C to conduct some industrial espionage. The espionage predicts that Drafts will choose the size of the advertising campaign with the following probabilities:

	Probability
D. chooses small	20%
D. chooses medium	10%
D. chooses large	70%

Importantly the espionage is accurate with the following probabilities (if the prediction is incorrect assume that the other two options are equally likely):

	Probability of being correct
Espionage predicts small	90%
Espionage predicts medium	95%
Espionage predicts large	70%

Use a decision tree to find the value of C that justifies using espionage.

4. Bob's utility function for his assets is $u(x) = \sqrt{x + 1000}$. Bob's car is valued at 7000. There is a 0.05 chance that Bob's car is destroyed over any given year. How much should Bob be willing to pay for an insurance policy that would replace his car if it were destroyed?
5. Consider the coin flipping example from the lecture notes: you are offered the opportunity of flipping a coin. If you choose to not flip you get a reward of 4 **thousand**. If you flip and the coin falls on *heads* you get a reward of 10 **thousand**. If you flip and the coin falls on *tails* you get nothing.

Use the general utility function $u : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ defined by:

$$u(x) = x^{\frac{1}{n}} \text{ for some } n \in \mathbb{Z}$$

to find a set of utility functions that give a risk-averse strategy. Does this hold if the rewards were 4 **million** and 10 **million**? What is the important factor that determines the value of n ?

6. I wish to buy a second hand car. There is one for sale at a local dealer for 1200, which comes with a 3 month warranty (any faults appearing in the first 3 months will be fixed for free). There is another for sale privately for 800, which appears to be of similar condition, but which has no warranty. I estimate the probability of a fault occurring in the first 3 months of ownership as .5, and the cost of repairing such a fault to be 600. To help me decide whether or not to buy the privately advertised car, I can get the Automobile Association (AA) to check the car for me, at a cost of 50. I estimate that:

$$P(\text{AA find fault} \mid \text{fault present}) = .8$$

$$P(\text{AA find fault} \mid \text{no fault present}) = 0$$

- Draw a decision tree representing the possible decisions and outcomes of this problem, with their respective costs and probabilities.
- Using a minimum expected cost criterion, decide whether I should buy from the dealer, buy privately, or get the AA to check the second hand car first.