1. (a) Provide definitions for the following terms:

- Normal form game.
- Strictly dominated strategy.
- Weakly dominated strategy.
- Best response strategy.
- Nash equilibrium.

[5]

(b) Consider the following game:

$$\begin{pmatrix} (2,\gamma) & (0,3) \\ (0,0) & (\gamma,1) \end{pmatrix}$$

(i) Prove that a pure Nash equilibrium exists for all values of $\gamma \in \mathbb{R}$.

[7]

(ii) State the Equality of Payoffs Theorem. Using this theorem obtain the values of γ (if it exists) for which the following (σ_1, σ_2) are mixed Nash equilibria for the game.

A. $(\sigma_1, \sigma_2) = ((1/2, 1/2), (1/4, 3/4))$
B. $(\sigma_1, \sigma_2) = ((1/2, 1/2), (2/3, 1/3))$
C. $(\sigma_1, \sigma_2) = ((1/5, 4/5), (7/9, 2/9))$

[13]

2. Consider the principal agent game: the first player is an employer and the second a potential employee. The employer must decide on two parameters:

A wage value: ω; A bonus value: B. The potential employee must then make two decisions:

- Whether or not to accept the job;
- If the job is accepted, of either putting in a high level or a low level of effort.

The success of the project that requires the employee is as follows:

- If the employee puts in a high level of effort the project will be successful with probability 1/2;
- If the employee puts in a low level of effort the project will be unsuccessful.

The monetary gain to the employer is as follows:

- If the employee does not take the job the utility is 0;
- If the project is successful: K;
- If the project is unsuccessful: κ.

The monetary gain to the employee is as follows:

- If the job offer is not accepted: 1;
- If the project is successful: $w + B - 1$ (the lost gain corresponds to the effort);
- If the project is unsuccessful despite a high level of effort: $w - 1$ (the lost gain corresponds to the effort), we assume that $w \geq 1$;
- If the project is unsuccessful because of a low level of effort: w.

We furthermore assume that the utility function of the employer is simply $u(x) = x$ while the utility function to the potential employee is of the form $u(x) = x^{\alpha}$ for $0 < \alpha < 1$ (i.e. the potential employee is risk averse).

(a) Give the extensive form representation for the above game. [5]

(b) List sensible assumptions with regards to the parameters and their interpretations. [4]

(c) Prove that the Nash equilibria for this game is the employer choosing $(w, B) = (1, 2^{1/\alpha})$ and the employee accepting the position. [14]

(d) Explain how α affects the bonus B offered by the employer. [2]

3. (a) Define a matching game.

[2]

(b) Define a blocking pair for a matching game.

[2]

(c) Define a stable matching for a matching game.

[2]

(d) Consider the following algorithm (for a matching game with suitors S and reviewers R):

(i) Assign every $s \in S$ and $r \in R$ to be unmatched

(ii) Pick some unmatched $s \in S$, let r be the top of s's preference list:
- If r is unmatched set $M(s) = r$
- If r is matched:
  - If r prefers s to $M^{-1}(r)$ then set $M(r) = s$
  - Otherwise s remains unmatched and remove r from s's preference list.

(iii) Repeat step 2 until all $s \in S$ are matched.

Prove that all possible executions of this algorithm yield the same stable matching and in this stable matching every suitor has the best possible partner of any stable matching.

[9]

(e) Obtain stable matchings for the following games:

	c: (A, B, C) •	• C: (a, b, c)
Game 1:	b: (B, C, A) •	• B: (a, c, b)
	a: (A, B, C) •	• A: (b, a, c)

[3]

	c: (C, A, B) •	• C: (b, a, c)
Game 2:	b: (B, C, A) •	• B: (a, c, b)
	a: (B, A, C) •	• A: (c, a, b)

[3]

	d: (A, B, D, C) •	• D: (d, a, b, c)
	c: (A, D, B, C) •	• C: (a, d, b, c)
Game 3:	b: (B, C, D, A) •	• B: (a, c, d, b)
	a: (D, A, B, C) •	• A: (d, b, a, c)

[4]