4. (a) Provide definitions for the following terms:

- Normal form game.
- Strictly dominated strategy.
- Weakly dominated strategy.
- Best response strategy.
- Nash equilibrium.

[5]

(b) Consider the following game:

$$\begin{pmatrix} (1,\alpha) & (0,2) \\ (0,0) & (\alpha,1) \end{pmatrix}$$

(i) Prove that a pure Nash equilibrium exists for all values of $\alpha \in \mathbb{R}$.

[7]

(ii) State the equality of payoffs theorem. Using this theorem obtain the value of α (if it exists) for which the following (σ_1, σ_2) are mixed Nash equilibria for the game.

A. $(\sigma_1, \sigma_2) = ((1/2, 1/2), (1/2, 1/2))$
B. $(\sigma_1, \sigma_2) = ((1/2, 1/2), (3/4, 1/4))$
C. $(\sigma_1, \sigma_2) = ((1/5, 4/5), (3/4, 1/4))$

[13]

5. (a) Define a (finitely) repeated game.

[4]

(b) Define a strategy in a repeated game.

[2]

(c) Prove that for any repeated game, any sequence of stage Nash profiles gives the outcome of a subgame perfect Nash equilibrium.

[7]

(d) For the following stage games, plot the possible outcomes for a repetition of $T = 2$ periods and obtain a Nash equilibria **that is not a sequence of stage Nash profiles**:

$$\begin{pmatrix} (3,4) & (1,2) & (2,5) \\ (-1,1) & (1,2) & (-1,-1) \end{pmatrix} \quad \begin{pmatrix} (3,1) & (1,1) \\ (-1,1) & (1,0) \\ (1,3) & (.5,1) \end{pmatrix}$$

$$\begin{pmatrix} (2,9) & (3,1) \\ (3,1) & (6,1) \\ (3,3) & (5,1) \end{pmatrix} \quad \begin{pmatrix} (1,1) & (1,0) & (1,1) \\ (1,2) & (3,2) & (2,5) \end{pmatrix}$$

[12]

6. (a) Define a stochastic game.

[4]

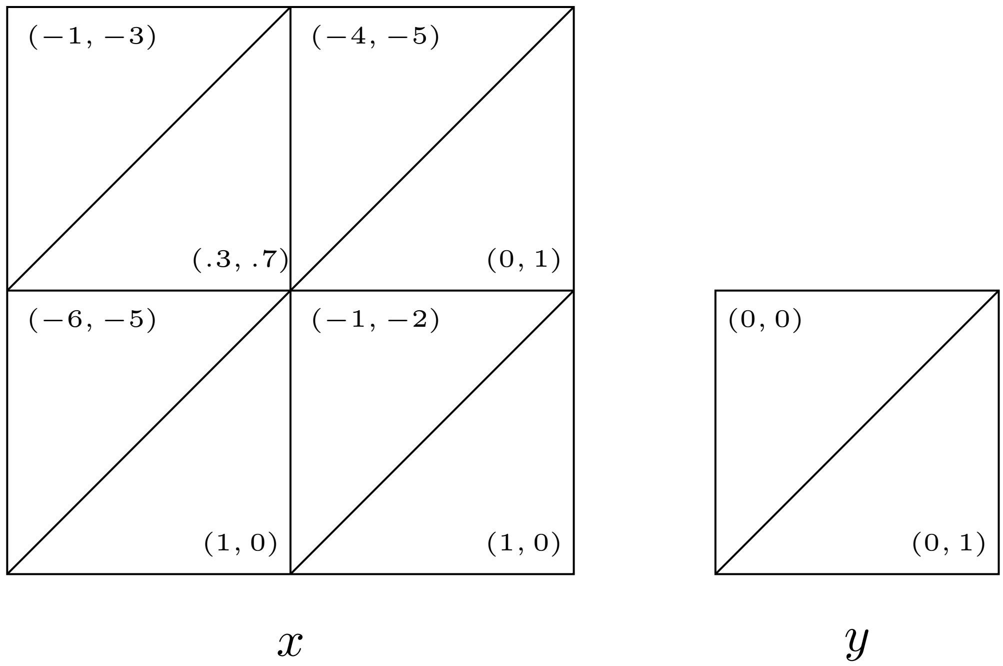
(b) Define a Markov strategy.

[2]

(c) Give the conditions for Nash equilibrium in a stochastic game.

[3]

(d) Obtain the pure strategy Nash equilibria (if any exist) for the following game with $\delta = .3$:



[16]